Improved Linear Group Detection for Combined Spatial Multiplexing/STBC Systems

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is shown that the optimum MMSE linear processing detector, by fully exploiting the Alamouti encoding pattern, can be designed avoiding any complex matrix inverses ($N_T \times N_T$ for arbitrary, yet even, $N_T$) by decomposing them into trivial $2 \times 2$ inverses.

We conclude this introduction with a notational remark: vector and matrices are denoted by lower- and upper-case bold letters, respectively. Superindices $^T$, $^*$ and $^H$ will be used to denote transpose, conjugate and hermitian, respectively, and $I_K$ is used to denote the $K$-dimensional identity matrix. The notation $A(i,\cdot)$ and $A(\cdot,i)$ will be used to refer to the $i$th row or column, respectively, of a matrix. Finally, we define the Alamouti transform of a $K \times 2$ matrix $x = [x_1 \ x_2]$ with $x_1, x_2$ being $K \times 1$ vectors as $A(x) \triangleq [-x_2^* \ x_1^*]$.

II. SYSTEM MODEL

We consider a system with $N_T$ and $N_R$ mutually uncorrelated transmit and receive antennas, respectively. As shown in Fig. 1, we assume that the transmitter simultaneously sends, using SDM, $N_s$ independent data streams and each stream is encoded according to the Alamouti scheme implying that $N_s = N_T/2$. Following the notation introduced in [9], [10], we define $h_n \triangleq [h_{(2n-1)}^T \ h_{(2n)}^T]$ with $n = 1, \ldots, N_s$ as the $N_R \times 2$ channel matrix associated with the $n$th data stream whose entries are independent zero-mean complex Gaussian variables with unit variance. Each column in $h_n$ denote the channel coefficients between each of the two transmit antennas associated to stream $n$ and the $N_R$ receiver antennas. We further assume that the channel remains static, at least, over a two-symbol block. For each data stream, a transmission matrix constructed according to the Alamouti encoding rule is defined as

$$S_n = \begin{bmatrix} s_{2n-1} & s_{2n} \\ s_{2n} & -s_{2n-1} \end{bmatrix}$$

where $s_{2n-1}$ and $s_{2n}$ denote two consecutive symbols of the $n$th data stream from an $M$-ary modulation alphabet. For completeness, we also define at this point $s_n = [s_{2n-1} \ s_{2n}]^T$ as the vector formed by the symbols to be transmitted over the $n$th data stream during a space-time block period. It is now possible to write the reception equation as:

$$y = HS + v = \sum_{i=1}^{N_s} h_i S_i + v$$

where $H = [h_1 \cdots h_{N_s}]$, $S = [S_1^T \cdots S_{N_s}^T]^T$ and $v$ is an $N_R \times 2$ noise vector whose entries are zero-mean complex Gaussian variables with variance $\sigma_n^2$.

III. EXISTING DETECTION SCHEMES

A. Group-based schemes

Fig. 1 shows that reception takes place in two steps: first, the block equaliser takes care of separating the different streams and then Alamouti decoding is applied on each stream independently. In the particular case of linear detectors, this operation can be formally expressed as $\hat{y}_n = W_n y$, where $W_n$ denotes the equalising matrix for the $n$th data stream and $\hat{y}_n$ is an $N_T \times 2$ matrix with the resulting $n$th stream data samples to be supplied to the Alamouti decoder. In [9], Zhao and Dubey proposed a linear group-based detection scheme for the specific case of $N_T = N_R = 4$, which corresponds to the DSTTD system with four receive antennas. In this case, the channel matrix $H$ can be rewritten as

$$H = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

where $A, B, C$ and $D$ are $2 \times 2$ matrices. The ZF group detectors for each of the two groups can easily be shown to be the rank 2 matrices

$$W_1^ZF = \begin{bmatrix} B^{-1} & -D^{-1} \\ B^{-1} & -D^{-1} \end{bmatrix}, \quad W_2^ZF = \begin{bmatrix} A^{-1} & -C^{-1} \\ A^{-1} & -C^{-1} \end{bmatrix}.$$

The rank deficiency of $W_1^ZF$ and $W_2^ZF$ allows a more compact expression [9]

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = W^ZF y = \begin{bmatrix} B^{-1} & -D^{-1} \\ A^{-1} & -C^{-1} \end{bmatrix} y$$

where $\hat{y}_1$ and $\hat{y}_2$ are used to denote the independent samples in $\hat{y}_1$ and $\hat{y}_2$, respectively.

It is well-known that ZF equalisation often leads to excessive noise enhancement and MMSE is usually the preferred criterion to design filters leading to lower BERs. The group-based MMSE equaliser can be shown to be [9], [10]

$$W_n^{MMSE} = h_n h_n^H \left( HH^H + \sigma_n^2 I_{N_R} \right)^{-1}.$$

The performance of both detectors, ZF and MMSE, could straightforwardly be improved by making use of the interference cancelation (IC) principle [9].

B. Direct detection schemes

In direct detection schemes, the receiver attempts to directly estimate the transmitted symbols without explicitly exploiting the structure of the Alamouti encoding. It is easy to check that the reception equation (1) can be rewritten as

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H \\ H_A \end{bmatrix} s + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Hs + \tilde{v},$$

where $y_1, y_2$ and $v_1, v_2$ are used to denote each of the individual columns of $y$ and $v$, respectively, $s = [s_1 \cdots s_{N_s}]^T$ and $H_A = [A(h_1) \cdots A(h_{N_s})]$. The reception equation in (4) is now in the standard form of MIMO systems and therefore, any of the standard MIMO detection techniques can be applied (e.g. ZF, MMSE, optimum combining (OC), IC-based, ML). Particularly important is the optimal receiver proposed in [10] based on ML detection as this has to be taken as the lower bound in terms of BER and the upper bound in terms of complexity for the rest of receivers. Two points are
worth noting with respect to (4): 1) proposed direct receivers ([6], [10], [14], [15]) are computed to recover the transmitted symbols and do not take into account the Alamouti structure and 2) the system matrix $H$ has dimensions $2N_T \times N_T$ and therefore, the matrix (pseudo)inverses and products usually required to compute the reception filters are substantially more complex than for group-based equalisers.

IV. IMPROVED GROUP-BASED SCHEMES

A. Whitening group-based detectors

The linear group-based detectors introduced in [9] do not take into account that the noise colouring introduced by the group equaliser causes the subsequent Alamouti decoding to lose its optimality. A simple improvement of these detectors is accomplished by complementing the equaliser with a noise-whitening filter. In the case of ZF, it is easy to see from (2) that the filtered noise has covariance $\Omega = W^ZF (W^ZF)^H \sigma_n^2$.

Since any covariance matrix is hermitian and positive definite, it is possible to obtain its eigen-decomposition $\Omega = VDV^H$, where $V$ is a matrix having as columns the eigenvectors of $\Omega$ and $D$ is a diagonal matrix containing the eigenvalues of $\Omega$ at its main diagonal, which can be shown to be positive and real. We can now define the zero-forcing noise whitening equaliser as

$$\hat{W}^ZF = (VD^{1/2})^{-1} W^ZF.$$  

The same procedure can be applied to the MMSE equalising matrices in (3) by using the corresponding covariance matrix for each group (e.g. $\Omega_n = W_n^{MMSE} (W_n^{MMSE})^H \sigma_n^2$) to obtain separated STBC streams affected by decorrelated noise samples.

B. Linear direct group-based detectors

We introduce a new class of linear receivers that we term direct linear group-based detectors and have the objective of achieving the same performance as linear direct detectors while reducing their computational complexity. The idea is to exploit the Alamouti structure to simplify the equaliser computation, which in turn implies performing the detection group-by-group. Starting from (4), the direct-estimation MMSE detector can be expressed as

$$\mathcal{W} = (\bar{H}^H \bar{H} + \sigma_n^2 I_{N_T})^{-1} \bar{H}^H$$

(5)

where, by exploiting the block structure of $\bar{H}$, the $2 \times 2$ matrices $\{\alpha_{ij}\}_{i,j=1}^{N_n}$ and $\{\beta_i\}_{i=1}^{N_n}$ can be defined as $\alpha_{ij} \triangleq \bar{h}_{ij}^H \bar{h}_j + (\bar{A}(h_i))^H H (h_j) = \alpha_{ij}^H$, and $\beta_i \triangleq \delta_{ii} + \sigma_n^2 I_2$, respectively. By expressing the filter solution $\mathcal{W}$ in $2 \times 2$ submatrices, Eq. (5) can be written as

$$\mathcal{W} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \\ \vdots & \vdots \\ W_{N_n,1} & W_{N_n,2} \end{bmatrix}$$

and

$$\begin{bmatrix} H_{11}^H & (A(h_1))^H \\ H_{21}^H & (A(h_2))^H \\ \vdots & \vdots \\ H_{N_n,1}^H & (A(h_{N_n}))^H \end{bmatrix}$$

with $\{X_{ij}\}_{i,j=1,2; i \geq j}$ being the solutions of the linear system of $2 \times 2$ matrix equations

$$\begin{bmatrix} I_2 & 0 & \cdots & 0 \\ 0 & I_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_2 \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1N_n} \\ X_{12}^H & X_{22} & \cdots & X_{2N_n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N_n,1}^H & X_{N_n,2}^H & \cdots & X_{N_n,N_n} \end{bmatrix} \times \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{N_n} \\ \beta_1 & \alpha_2 & \cdots & \alpha_{N_n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1 & \beta_2 & \cdots & \beta_{N_n} \end{bmatrix}.$$  

Now, thanks to the Alamouti encoding pattern it is shown in the Appendix that $\mathcal{W}_{j2} = (A(W_{j1}^T))^T$, allowing the estimated symbols to be computed as

$$\begin{bmatrix} \hat{s}_1 \\ \vdots \\ \hat{s}_{N_n} \end{bmatrix} = \mathcal{W}_{y} = \begin{bmatrix} W_{11} y_1 + (A(W_{11}^T))^T y_2^* \\ \vdots \\ W_{N_n,1} y_1 + (A(W_{N_n,1}^T))^T y_2^* \end{bmatrix}.$$  

The detection procedure given by (6) can be put in the form of group-based detection as described in Section III-A by defining the group equaliser as $\mathcal{W}_n = [\mathcal{W}_{n1} (A(W_{n1}^T))^T]$. Now the symbol estimates for the $n$th group can be computed as $\hat{s}_n = \mathcal{W}_n y$.

The attractive feature of the linear-direct group-based detector is that for the computation of $\mathcal{W}$ only trivial inverses of $2 \times 2$ matrices are required rather than inverses involving $N_T \times N_T$ matrices as required in the direct solution of (5). As an example, assuming $N_T = 4$ and irrespective of the value of $N_R$, it is straightforward to show that

$$\mathcal{W} = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^H & W_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^H & X_{22} \end{bmatrix} \bar{H}^H$$

where $X_{12} = (\alpha_{12}^H - \beta_2 \alpha_{11} \beta_1)^{-1}$, $X_{11} = -X_{12} \beta_2 \alpha_{12}^H$ and $X_{22} = -X_{12} \beta_1 \alpha_{12}^H$.

The Alamouti structure can also be used to draw some conclusions regarding the diversity order achieved by this detector. The estimated symbols (e.g. prior to detection) are given by

$$\hat{s} = \mathcal{W} \hat{H} s + \hat{\mathcal{W}} \hat{v}.$$  

(7)

The detection of each symbol is then performed independently by slicing the estimated symbol, $\hat{s}_i = \mathcal{W}(i, \cdot) \hat{H}(\cdot, i) s + \mathcal{W}(i, \cdot) \hat{v}$, to its nearest constellation point $\hat{s}_i$. 
Without loss of generality, we focus now on the first symbol of the first STBC stream, $s_1$. It is easy to check that thanks to the Alamouti encoding pattern it holds
\[
\tilde{s}_1 = \tilde{\mathbf{W}}(1, \cdot) \tilde{\mathbf{H}}(\cdot, 1)s + \tilde{\mathbf{W}}(1, \cdot)\mathbf{W}\bar{v},
\] (8)
where $\tilde{\mathbf{H}}$ is a $2N_R \times (N_T - 1)$ matrix given by
\[
\tilde{\mathbf{H}} = [\mathbf{H}(\cdot, 1) \mathbf{H}(\cdot, 3) \ldots \mathbf{H}(\cdot, N_T)]
\]
and the corresponding MMSE equaliser is computed as
\[
\tilde{\mathbf{W}} = (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \sigma_n^2 \mathbf{I}_{N_T-1})^{-1} \tilde{\mathbf{H}}^H.
\]
In other words, since $s_2$ is orthogonal to $s_1$, the channel and equaliser coefficients affecting $s_2$ need not be taken into account when detecting $s_1$. Consequently, to nearly all effects, and from the point of view of detecting $s_1$, the situation is analogous to the optimum combining scenario addressed in [16] by considering the system to have $2N_R$ (receive) antennas and $N_T - 1$ co-channel interferers. Therefore, as stated by Winters et al., the diversity order of this scheme is $2N_R(N_T - 1) + 1$. We note that exact bit error rates for this scenario can be found in [17] for the case of uncorrelated channel coefficients. The hybrid SDM/STBC setup considered here cannot be perfectly fitted into the model of [17], since not all channel entries in $\tilde{\mathbf{H}}$ are independent.

V. NUMERICAL RESULTS AND DISCUSSION.

We consider a hybrid SDM/STBC system with $N_T = N_R = 4$ and $N_s = 2$ (DSTTD setup) transmitting over a frequency non-selective block fading channel using BPSK modulation with unit power symbols (e.g. $E_b/N_0 = 1/2\sigma_n^2$). For comparison purposes, we have also considered the performance of single stream $2 \times 2$-STBC and $2 \times 4$-STBC systems, both configurations having half the transmission rate of the DSTTD setup. Fig. 2 summarises the BER results of the different algorithms and configurations. The first noticeable fact, albeit expected, is that the $2 \times 4$-STBC constitutes the BER lower bound for all DSTTD configurations. Remarkably, the ML-based detection in DSTTD as proposed in [10] attains this lower bound, implying that when using this detector, there is no performance penalty in transmitting at a double rate. The MMSE and ZF group-based detectors proposed in [9] are seen to offer a poor performance in comparison with the direct-estimation MMSE receiver. The noise whitening step we have introduced in Section IV-A significantly improves the performance of these detectors (Group WZF, Group WMMSE) but they are still far from the more computationally complex direct-estimation algorithms. It is interesting to notice that the noise-whitened group-based ZF equaliser has exactly the same performance as the $2 \times 2$-STBC system implying that the introduced decorrelation procedure effectively transforms the DSTTD system into two parallel non-interfering $2 \times 2$-STBC systems. Finally we show the results for the direct-estimation MMSE detector and our group-based direct solution proposed in Section IV-B. Obviously, the two solutions attain exactly the same BER although our proposal has much lower computational complexity.

In Fig. 3 the performance of the proposed group-based direct-estimation MMSE in comparison with ML-based detection is presented for different number of receive antennas. (Whitened) Group-based detectors are not included in this comparison due to their requirement of $N_R \geq N_T$. This figure illustrates some interesting points regarding the diversity achieved by the various configurations. In the case of ML detection, it can be observed that, as expected, the attained diversity order (e.g. asymptotic slope of the BER curve) is $2N_R$ (the two factor comes from the Alamouti code). This effect is more evident for $N_R = 1$ and $N_R = 2$ since for higher number of antennas the asymptotic diversity is achieved at BER levels typically out of the range of interest [18]. For the case of the MMSE direct group detector, it is observed that the attained diversity order is indeed $2(N_R - 1)$ in accordance with the results in [16], which implies that the MMSE detector requires one additional receive antenna to achieve the same diversity order as the ML receiver.
VI. CONCLUSIONS

In this letter, we have introduced two new families of detectors for combined SDM/STBC systems, namely, the whitening group-based detectors and the linear direct group-based detectors. While the first family improves the performance with respect to previous group-based detection approaches, their performance still lags behind that of the more complex direct detectors. Our second proposal decomposes the linear direct solution into (smaller dimension) group-based solutions by exploiting the Alamouti encoding structure, reducing in this way the overall computational complexity while attaining the same BER performance.

APPENDIX

Proof that \( \mathcal{W}_{j2} = \left( A \left( \mathcal{W}_{j1}^T \right) \right)^T \)

In this appendix, we define an Alamouti matrix as an invertible \( 2 \times 2 \) matrix \( S \) such that

\[
S = \begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix} = \begin{bmatrix}
s_{11} & s_{12} \\
-s_{12} & s_{11}
\end{bmatrix}.
\]

It is easy to check that the set of Alamouti matrices is closed under the sum, product and inverse operations.

We say that a \( 2N \times 2M \) matrix

\[
S = \begin{bmatrix}
s_{11} & \cdots & s_{1M} \\
\vdots & \ddots & \vdots \\
s_{N1} & \cdots & s_{NM}
\end{bmatrix}
\]

is block Alamouti if it is invertible and all its \( 2 \times 2 \) constituent matrices \( S_{nm} \) are Alamouti. The closure property of Alamouti matrices makes it obvious that the set of block Alamouti matrices is also closed under the sum and product operations. Using this closure property, let us now resort to mathematical induction to prove that the set of \( 2N \times 2M \) block Alamouti matrices is also closed under inversion operation:

- For \( N = 1 \), the block Alamouti matrix is also Alamouti and, in consequence, its inverse is also block Alamouti.
- Now, assuming that the closure property under inversion holds for \( N = k - 1 \), we will prove it for \( N = k \). Let

\[
S_k = \begin{bmatrix}
s_{11} & \cdots & s_{1k} \\
\vdots & \ddots & \vdots \\
s_{k1} & \cdots & s_{kk}
\end{bmatrix},
\]

be a \( 2k \times 2k \) block Alamouti matrix. By defining \( \Omega_k \triangleq [S_{1k} \ldots S_{(k-1)k}] \) and \( \Delta_k \triangleq [S_{1k} \ldots S_{(k-1)k}]^T \), it follows that

\[
S_k = \begin{bmatrix}
S_{k-1} & \Delta_k \\
\Omega_k & S_{kk}
\end{bmatrix},
\]

and using the block matrix inversion formula [19] we obtain

\[
S_{k}^{-1} = \begin{bmatrix}
C_1^{-1} & -S_{k-1}^{-1} \Delta_k C_2^{-1} \\
-C_2^{-1} \Omega_k S_{k-1}^{-1} & C_2^{-1}
\end{bmatrix},
\]

with

\[
C_1 \triangleq S_{k-1} - \Delta_k S_{kk}^{-1} \Omega_k, \\
C_2 \triangleq S_{kk} - \Omega_k S_{k-1}^{-1} \Delta_k.
\]

By the closure property of Alamouti matrices under inversion operation and the closure property of block Alamouti matrices under sum and product operations, it follows that \( C_1 \) is a \( (2k - 1) \times (2k - 1) \) block Alamouti matrix. As we have assumed that the closure property under inversion holds for \( 2(k - 1) \times 2(k - 1) \) block Alamouti matrices, then \( C_1^{-1} \) and \( S_{k-1}^{-1} \) are block Alamouti matrices and accordingly, it is now easy to check that \( S_{k}^{-1} \) is a \( 2k \times 2k \) block Alamouti matrix, which completes the induction proof.

An easy computation shows that \( \{x_{ij}\}_{i,j=1}^{N_s} \) and \( \{\beta_i\}_{i=1}^{N_s} \) are Alamouti matrices. Therefore, on account of the closure property of square block Alamouti matrices under the inversion operation, we have that \( \{X_{ij}\}_{i,j=1}^{N_s} \) also belong to the set of Alamouti matrices, and thus,

\[
\chi_{ij} = \begin{bmatrix}
x_{ij11} & x_{ij12} \\
-x_{ij12} & x_{ij11}
\end{bmatrix}.
\]

Using this expression, it follows that

\[
\mathcal{W}_{i1} = \sum_{j=1}^{N_s} \chi_{ij} h^H_j
= \sum_{j=1}^{N_s} \left[ x_{ij11} h^H_{j(2j-1)} + x_{ij12} h^H_{j(2j)} \right] + x_{ij12} h^H_{j(2j-1)} - x_{ij11} h^H_{j(2j)}
= \chi_{ij} (h_j)^T,
\]

and

\[
\mathcal{W}_{i2} = \sum_{j=1}^{N_s} \chi_{ij} \left( A \left( h_j \right) \right)^T
= \sum_{j=1}^{N_s} \left[ -x_{ij11} h^H_{j(2j)} + x_{ij12} h^H_{j(2j-1)} \right] - x_{ij12} h^H_{j(2j)} + x_{ij11} h^H_{j(2j-1)}
= \chi_{ij} \left( A \left( \mathcal{W}_{i1} \right) \right)^T,
\]

which concludes the proof.

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